Erythrocytoric and Related

Haemoviscology

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Artificial Organs BE6810Y82

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1. Introductory Comments

Blood, being a fluid with suspended particles, may be postulated as following several principles common to such fluids. In high shear we observe the Fahraeus-Lindqvist, or axial accumulation, effect. With lower shear we have rouleau formations, with cells flowing as a sack of coins, united by some yet unknown adhesive. The cell often deforms in small vessels, where a hydrodynamic lubrication model may be appropriate for flow

$$q_x = [-(h^3/12/(\mu))(dp/dx) - uh/2]$$

[1] For rigid spheres Einstein obtained

$$\mu_p / \mu_0 = 1+5c/2$$

$$|c<.18; (\mu^{-1}-)(dv/d\xi)|c<.12$$

But for small spherical droplets, G.I. Taylor replaced the concentration's multiplier by

$$((\mu_0 + 5\mu_*/2)/(\mu_0 + \mu_*)),$$

which may be multiplied by a geometric factor which surpasses unity for assymetric particles.[2] A correlation has been determined for yield stress

$$\tau_0 = [(HCT-.1)(C_f + .5)]^2 H(HCT-.1)$$

$$|C_f \in [.21,.46] \; \& \; HCT).1,$$

with C_f representing fibrinogen concentration and HCT the hematocrit, or volume ratio of solids in blood, which is usually

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[1] p.174, Fuller, Duley. D., Theory and Practice of Lubrication for Engineers, New York: John Wiley (1956).

[2] pp.155-157, Caro, C.G., Pedley, T.J., Schroeter, R.C., & Seed, W.A., The Mechanics of the Circulation, Oxford(1978); Weinbaum, S., Biofluid Heat and Mass Transfer, Fall Semester, 1981 Course in The City College of the City University of New York, ME5506.

2. Erythrocyte Rouleau Aggregation Phenomenon

Normal whole blood, being a heterogenous fluid, has red cells which aggregate at low shear rates, forming "rouleaux", erythrocytes which are assembled as a roll of coins [see figure.] This is believed to result in viscosity increases. This is important for many normal and pathological situations.

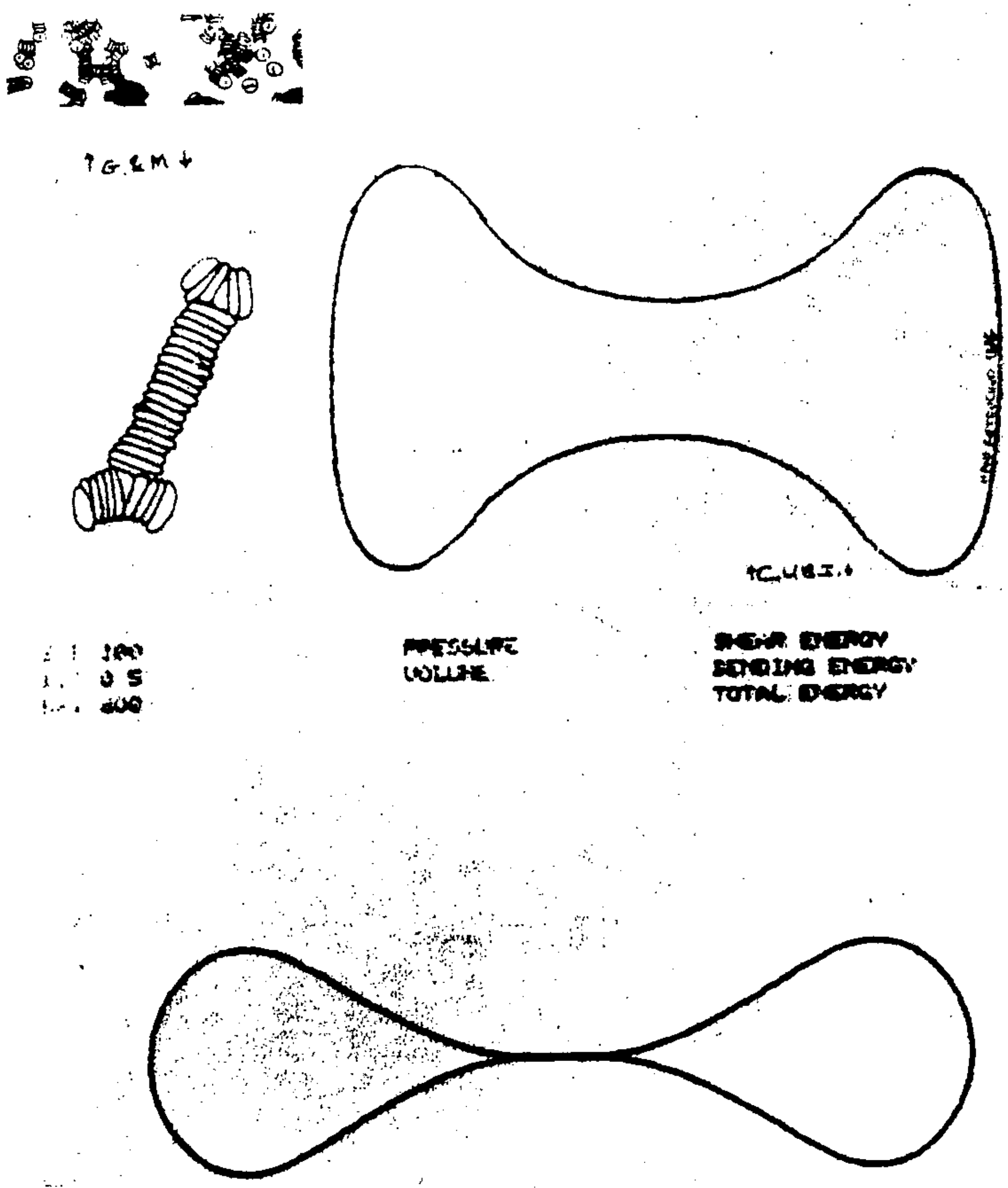

Figure 2-1: Rouleau Phenomenon,
[Goldsmith & Mason <u>opere</u> <u>citato</u>;
Unpublished Computer Approximations,
Columbia Bioengineering Institute,
all rights reserved by institute.]

3. Fahraeus-Lindqvist Effect

Robin Fahraeus and Torsen Lindqvist from the Pathological Institute in Uppsala, Sweden stated

> Nearly one hundred years have passed since the French physician Poiseuille[10] took up for consideration the important problem of the resistance of the bloodstream in the narrow parts of the vascular system. As experimental difficulties arose with blood, his fundamental investigations were confined to experiments with water and different fluids in glass capillaries. He found, as is well known, that the time of efflux of a given volume of fluid is directly as the length of the tube, inversely as the difference of pressure at the two ends and inversely as the fourth power of the diameter. ....with water... agreed very well with the law of Poiseuille...The law of Poiseuille does not apply to the flow of blood in capilary tubes of a diameter below about 0.3 mm.[11]

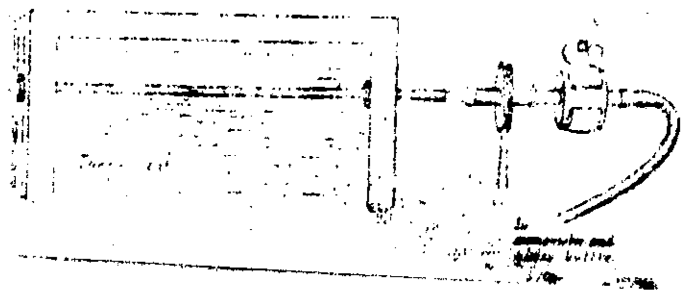

Figure 3-1: Aparatus of Fahraeus and Lindqvist, [opere citato.]

This is known as the Fahraeus-Lidqvist or axial accumulation effect. This effect occurs only at high rates of shear. It may be related to the occurance of a cell-free layer near the tube wall. This has been verified via high-speed photography of blood flow in glass tubes (in vitro) and in blood vessels of mammals (in vivo.) Blood accumulates at the axis so pronouncedly that if blood is not drawn from the center of the vessel, the hematocrit, or volume fraction of cells in blood, may differ by a fourth.[12] [13] This may be explained via the cell-free marginal layer model

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[10] Poiseuille, Annales de Chimie et de Physiol., vii, 50[1843]; Compt. Rend. d. Seances De L'Acad. Sci., xvi, 60[1843]

[11] R Fahraeus, & T Lindqvist, The Viscosity of Blood in Narrow Capillary Tubes, Am.J.Physiol., 96, [1931], pp.562-568.

[12] Cooney, opere citato, pp.54-58.

[13] This axial accumulation could limit interaction of the cells to their surroundings, such as the transport of oxygen and carbon dioxide.

4. Effect on Artificial Organs

Analysis of red cells is necessary for correct design of internal and external artificial organs. Since erythrocytes may behave unusually, certain conditions must be compensated for or avoided in design. At high rates of shear we have the Fahraeus-Lindqvist effect. While, at low rates of shear, we have rouleaux formation. Both can be detrimental and thus the median shear rates must be chosen. Artificial organs include external devices such as heart-lung and kidney-dialysis machines, as well as internal organs such as artificial kidneys and hearts.[16]

The Fahraeus-Lindqvist Effect,[17] may be accounted for under certain conditions. In the words of Professor Leonard, ~~who has just completed his three-year chairmanship of chemical engineering here at Columbia,~~ and a student of his: "the effects of particle shape, deformability, and interaction with other particles must be considered before predictions of the radial drift phenomenon can be accurate enough quantitavely to be of real value in the design of artificial organs and plasma cell separation devices."[18]

The effect of rouleaux on artificial organs is as follows: Normally, in a small vessel, a small difference in pressure (.02 torr) is sufficient to induce flow (e.g.:capillary 50 μ in diameter and 0.5 cm. in length). If, however there are 500 such vessels in a system, 10 torr would be needed to restart halted flow. This would result in injury due to lack of oxygen, which is transported by erythrocytes.[19] This must thus be compensated for in design.

Perhaps the ideas of this paper are best concluded by this rather lengthy quote from artificial heart researchers at Goodyear:

> Considering that more people die from cardiovascular related causes than all other causes combined, there seems to be little question but what [sic] the artificial heart, when fully developed, will be regarded as <u>the</u> major

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16 Cooney, <u>opere citato</u> ,pp.302-438.

17 also descriptively called radial drift, or axial accumulation -- accumulation at the center of a vessel

18 R V Repetti, & E F Leonard, Physical Basis for the Axial Accumulation of Red Blood Cells, in <u>Chemical Engineering in Medicine</u>, <u>Chemical Engineering Progress Symposium Series</u>, E.F.Leonard, ed., 62, no.66, [1966], pp.86.

19 Middleman, <u>opere citato</u>, pp.83-84.

5. Haemoviscological Computations

5.1 Correlation From Experimental Data

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LOG FILE IS MLAB.LOG.
LEAVE THIS SESSION WITH THE COMMAND 'EXIT'.
*FCT SQ(X)=X^2
*SQRTAU=LIST(.001,.2,.3,.45,.55,.75,.85)
*SQRTRATE=LIST(0,.2,.25,.5,.7,1.5,2.2)
*DATA COL 1= SQ ON SQRTRATE;DATA COL 2= SQ ON SQRTAU
*FCT  STRESS(RATE)=MU*RATE^POWER+INISTRESS
*INISTRESS=0;MU=.3;POWER=.77
*FIT (MU,POWER,INISTRESS), STRESS TO DATA
[...DISCONTINUED FOR BREVITY....]
BEGIN ITERATION 4
BEGIN ITERATION 4.1, SUM OF SQUARES= .4492900-2

FINAL                 NORMAL ERROR       DEPENDENCY
PARAMETER VALUES:     STANDARD ERRORS:   VALUES:
    .402009            .4144050-1        .901354     MU
    .410949            .4810160-1        .766126     POWER
  -.2465130-1          .3097880-1        .832861     INISTRESS

CONVERGED
RMS ERROR= .3350830-1
FINAL SUM OF SQUARES= .4491210-2
# OF ITERATIONS USED=4
*TTYDRAW DATA &'( STRESS ON DATA COL 1)
```

$$\tau = \mu \left(\frac{\partial v}{\partial \xi}\right)^n + \tau_0$$

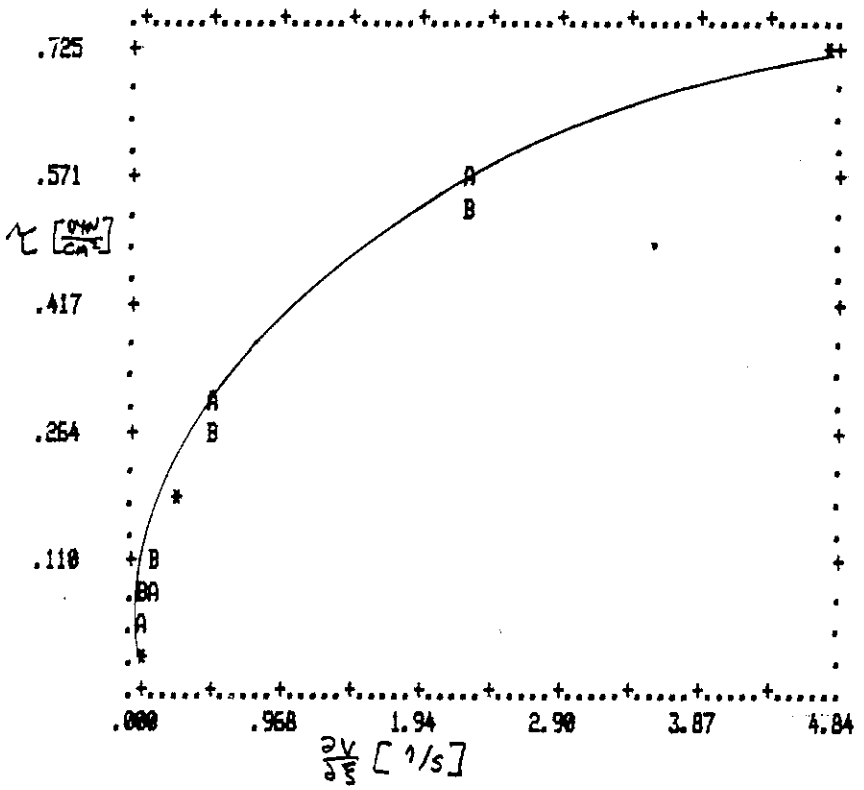

Figure 5-1: Haemoviscology Curve-Fitting Computations

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LOG FILE IS MLAB.LOG.
LEAVE THIS SESSION WITH THE COMMAND 'EXIT'.
*FCT STRESS(RATE)=.402009*RATE^.410949 -.246513@-1
*FCT NEWTSTRESS(RATE)=MU*RATE+INISTRESS
*INISTRESS=2;MU=.01
*FIT(MU,INISTRESS),NEWTSTRESS TO POINTS(STRESS,100:800)
[....DISCONTINUED FOR BREVITY....]
FINAL               NORMAL ERROR       DEPENDENCY
PARAMETER VALUES:   STANDARD ERRORS:   VALUES:
    .485191@-2        .262935@-4        .831793    MU
    2.59994           .129734@-1        .831793    INISTRESS

CONVERGED
RMS ERROR= .140875
FINAL SUM OF SQUARES= 13.8723
# OF ITERATIONS USED=2
*TTYDRAW POINTS(NEWTSTRESS,100:800)&'(STRESS ON 100:800)
```
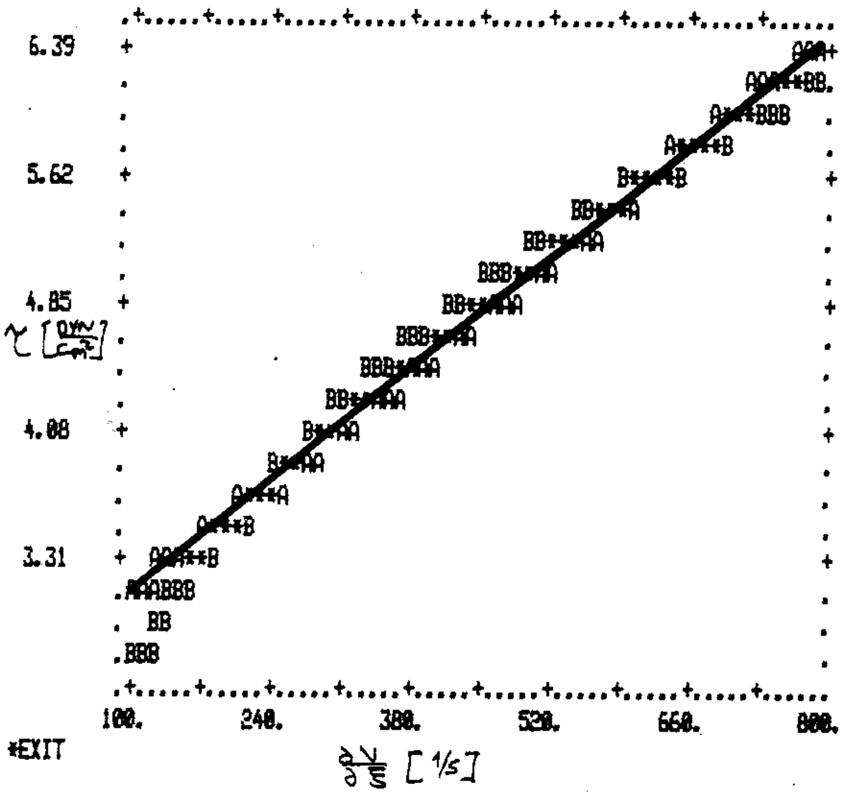

$\tau = \mu \left(\frac{\partial v}{\partial \xi} \right)^1 + \tau_0$

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*EXIT
```

Figure 5-2: Haemoviscological Linearisation for Artificial Organ Shear Rates

viscosity of water is given by the Columbia Physiology Syllabus[26] Caro[27] indicates shear rates of 1000/s as typical in vivo, at which he states the insignificance of non-Newtonian properties, and a viscosity of 3-4mNsm^{-2}. The linear model

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26 Physiology Syllabus, Columbia University in the City of New York, College of Physicians and Surgeons(1981), 5ed., p.199.

27 opere citato, p.176.

15JUL

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VP:OS\CUBI>> ty capLRY.DAT.53
"Columbia University Bioengineering Institute
  Vasos-Peter John Panagiotopoulos II
  MLAB DAT DO file for capillary
  15JUL81";
type "[CUBI CAPLRY]";
fct nu(alpha)= (   (1+lambda+alpha)/(1-lambda-alpha)
                 - (1+lambda-alpha)/(1-lambda+alpha)
               )
              /(   (1+lambda+alpha)/(1-lambda-alpha)
        + (1+lambda-alpha)/(1-lambda+alpha)
        + eta/lambda*(1+lambda)
               );
fct k(alpha)=1/(1+lambda)
               *(   (1+nu(alpha))/(1-lambda+alpha)
                  +(1-nu(alpha))/(1-lambda-alpha) );
fct q(alpha)=1/k(alpha)*(1-lambda+alpha*nu(alpha))
             +1/12*(   (1-lambda+alpha)^3+(1-lambda-alpha)^3);
lambdary=list(.01,.25,.5,.75,.85,.9,.95,.98);
etary=list(0,1,10,100);
for ilambda=6:8 do
(lambda=lambdary(ilambda);
         for ieta=1:4 do
          (eta=etary(ieta);
           alphary=0:(1-lambda-.01):((1-lambda)*.01);
           draw points(q,alphary)
          );
         type "Prepare Plotter:";
          do "tty:";
         display=t4010;
          type lambda;
  string "Q^(^B)", at .05,140;
         do "tty:"; "wait for terminal command";
          display=t4010;
 do "tty:";
          delete defaultwindow;
    delete defaultwindow;
  type "new plot"
        );
VP:OS\CUBI>> tsp 4800
!n
```